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# Unified numerical predictor-corrector guidance based on characteristic model

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**Abstract**: Aerocapture is one of the key technologies for low-cost transportation, with high demands of autonomy, accuracy, and robustness of guidance and control, due to its high reliability requirements for only one chance of trying. A unified numerical predictor-corrector guidance method based on characteristic models for aerocapture is proposed. The numerical predictor-corrector guidance method is used to achieve autonomy and high accuracy, and the characteristic model control method is introduced to achieve robustness. At the same time, by transforming path constraints, characteristic model equations including apogee deviation and altitude differentiation are established. Based on the characteristic model equations, a unified guidance law which can satisfy path constraints and guidance objectives simultaneously is designed. In guidance problems, guidance deviation is not directly obtained from the output of the dynamics at present, but is calculated through integral and algebraic equations. Therefore, the method of directly discretizing differential equations cannot be used to establish characteristic models, which brings great difficulty to characteristic modeling. A method for characteristic modeling of guidance problems is proposed, and convergence analysis of the proposed guidance law is also provided. Finally, a joint numerical simulation of guidance and control considering navigation deviation and various uncertainties is conducted to verify the effectiveness of the proposed method. The proposed unified method can be extended to general aerodynamic entry guidance designs, providing theoretical and methodological support for them.

**Keywords**: aerocapture; path constraint; characteristic model; unified numerical predictor-corrector guidance; convergence

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## 0 Introduction

As an inevitable approach for the mankind to exploring the mysteries of the universe, deep space exploration is to explore the wider space of the solar system on the basis of the achieved breakthroughs in near-Earth space activities. Here, low-cost space transport is one key technology. The use of aerodynamic re-orbiting techniques allows for increased payload mass in many re-orbiting missions in the atmosphere, and hence aerodynamic-assisted re-orbiting will yield impressive cost reductions in spacecraft design. For planets with atmosphere, aerodynamic-assisted re-orbiting can save large amounts of propellant. For those without atmosphere (e. g., the Moon and Mercury), braking can still be accomplished using aerodynamic forces, which activity is generally

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required for the return entry into Earth's low orbit from an interplanetary voyage. Aerocapture refers to the use of aerodynamic forces generated by planetary atmospheres to reduce orbital energy and achieve the transfer from hyperbolic orbits to target circular orbit. The concept of aerocapture was proposed by London in the 1960s and its feasibility was analysed [1]. Aerocapture has considerable fuel-saving advantages over traditional chemically-fueled impulse re-orbiting, and the Space Propulsion Technology Program Office at NASA's Marshall Space Flight Center has conducted a documented study of the advantages and disadvantages of both aerobraking and aerocapture, with systematic evaluation of their risks. The results of the study, based on simulations of an Earth return capsule carrying four 890 N orbital manoeuvre engines, concluded that the engine operation time for pure thrust braking and aerocapture were 51 and 21 minutes respectively (where the operation time for aerocapture is for the trajectory adjusting from formatted ellipse orbit to the target orbit). This is equivalent to a 40 percent saving in fuel. If the vehicle carries a 490 N small-to-medium-thrust orbital engine, the percentage of fuel saved will be further increased, which will be significant in future re-entry missions. Studies have shown that the use of aerodynamically assisted re-orbiting (Mars aerocapture and Earth aerocapture on re-entry) is the key to mission realisation on a human mission to Mars if current propulsion technologies are used. On lunar re-entry, if the return capsule returns to Earth by adopting the aerodynamic assisted re-orbiting technology, a large amount of propellant can be saved with the take-off mass in the near-Earth orbit reduced by 16% to 20%. Therefore, aerodynamic power assisted reorbiting is a key technology in the design of advanced launch vehicles. However, unlike the successfully implemented aerobraking technology, the aerocapture technology is still at research and ground test stage.

The task of guidance is to enable the spacecraft to fly in an excellent thermodynamic environment and to accurately meet various constraints such as overload, thermal and terminal conditions. The aerocapture mission has only one chance to be realised and requires very high reliability, thus placing high demands on the accuracy and robustness of guidance and control. Reviewing the progress of research and application of re-entry guidance algorithms, the theory of re-entry guidance has been studied since the 1950s and has gone through the development stages of Apollo guidance algorithms, Space Shuttle guidance algorithms, and numerical predictor-corrector guidance. With the significant improvement of computer technology, the numerical predictor-corrector guidance method has made great progress after more than 60 years of development. In the case of a large initial error and issues with the vehicle, this method can still achieve good guidance accuracy. The aerocapture guidance problem follows the same development process, dating back to the late 1970s, when the accuracy and orbital characteristics of intra-atmospheric navigation guidance and control for aerocapture missions were analyzed[2]. Based on the Space Shuttle guidance law, a hybrid predictor-corrector guidance method was proposed[3], which is essentially an analytical guidance law based on the linearisation of small perturbations, and is the basis of the research field of analytical guidance for aerocapture[4-5]. However, for the aerocapture problem of Mars which has strong uncertainty in aerodynamic environment, the analytical guidance method can hardly meet the requirements of guidance accuracy and robustness, and hence the numerical predictor-corrector method has gradually become the main research direction for solving the aerodynamic deorbiting problem[6]. The control quantities studied above are all studies of single control quantity, i. e., the bank angle. In order to further improve the control capability, an aerodynamic capture guidance method was proposed using two control quantities, the bank angle and the angle of attack, to control the aerodynamic flight trajectory[7].

The trajectory constraints are the key problem for the numerical predictor-corrector guidance law[8-11]. It is pointed out that for a small lift-to-drag ratio vehicle, the peak overload of the process is smaller when the initial bank angle amplitude of the vehicle is larger[8]. Based on this, an overload eliminator is designed to predict the peak overload when the vehicle enters the atmosphere and increases the initial bank angle if it is too large. The trajectory is divided into two parts and the vehicle is guided in the first part with a constant value thermofluid trajectory and in the second part with an unconstrained predictor-corrector guidance algorithm[9]. A lift-up guidance command is employed when the vehicle overload reaches the limit[10]. The trajectory constraints are corresponded to the drag to which the vehicle is subjected to obtain the range allowed by the drag[11]. When the drag reaches the boundary a strategy to increase the vehicle altitude is used to reduce the drag. Convergence is also an issue to be considered for numerical prediction of the correction guidance law, and a convergence analysis is carried out in the optimisation framework [12].

Unlike the method using optimisation in [12], the numerical predictor-corrector guidance method based on characteristic model extends the control method to guidance, which has stronger robustness compared with the optimisation method, but the method in control cannot be used in the performance analysis. This is due to the fact that, for the aerocapture problem, the numerical predictor-corrector guidance method first integrates the dynamics at the current moment up to the atmospheric exit, then uses the orbital dynamics formula to obtain the value of the transition orbit apogee, and finally its difference from the desired apogee is used for the feedback of the guidance law. It can be seen that, unlike the control design, the difference used in guidance is not the difference between the state of the differential equation and the reference input, and the error feedback in guidance is the result of the integration and algebraic equation calculation, so it cannot be analysed directly in the same way as the performance analysis in the field of control. An alternative approach is hence needed.

In this paper, a unified numerical predictor-corrector guidance method based on characteristic models for aerocapture is proposed, with also the convergence analysis and joint numerical simulation of guidance and control.

# 1 Problem description

Consider the following spacecraft dynamics[6]:

$$\left.\begin{aligned}
&\dot{r}=V\sin\gamma,\ \dot{\theta}=\frac{V\cos\gamma\sin\psi}{r\cos\varphi},\ \dot{\varphi}=\frac{V\cos\gamma\cos\psi}{r}\\
&\dot{V}=-\frac{D}{m}-\frac{\mu\sin\gamma}{r^2}+\Omega^2 r\cos\varphi(\sin\gamma\cos\varphi-\\
&\quad\cos\gamma\cos\psi\sin\varphi)\\
&V\dot{\gamma}=\frac{L\cos\sigma}{m}+\left(\frac{V^2}{r}-\frac{\mu}{r^2}\right)\cos\gamma+2\Omega V\cos\varphi\sin\psi+\\
&\quad\Omega^2 r\cos\varphi(\cos\gamma\cos\varphi+\sin\gamma\cos\psi\sin\varphi)\\
&V\dot{\psi}=\frac{L\sin\sigma}{m\cos\gamma}+\frac{V^2}{r}cos\gamma\sin\psi\tan\varphi-\\
&\quad 2\Omega V(\tan\gamma\cos\psi\cos\varphi-\sin\varphi)+\\
&\quad\frac{\Omega^2 r}{\cos\gamma}\sin\psi\sin\varphi\cos\varphi
\end{aligned}\right\}\tag{1}$$

where $\mu$ denotes the gravitational constant of target stars (Mars, Venus, Titan, etc. with atmosphere); $m$ denotes the mass of spacecraft; $r$ denotes the distance from the spacecraft to the centre of mass of the target stars; $\theta$ and $\varphi$ denote latitude and longitude respectively; $V$ denotes the relative velocity; $\gamma$ and $\psi$ denote the flight path angle and heading angle respectively; $\sigma$ denotes the bank angle, which is an input for the guidance; $\Omega$ denotes the angular velocity of the target stars; $L$ and $D$ denote the aerodynamic lift and drag respectively,

$$L = \frac{S_{\mathrm{ref}} C_{\mathrm{L}} \rho V^2}{2}$$

$$D = \frac{S_{\mathrm{ref}} C_{\mathrm{D}} \rho V^2}{2}$$

$S_{\mathrm{ref}}$ denotes the spacecraft reference area; $C_{\mathrm{L}}$ and $C_{\mathrm{D}}$ denote the lift and drag coefficients respectively; $\rho$ denotes the atmospheric density,

$$\rho = \rho_0 \exp\left(\beta_{\mathrm{r}} \frac{h}{h_0}\right)$$

$$h = r - R_0$$

$\rho_0$ is the atmospheric density at the reference altitude $h_0$, and $\rho_0$ and $h_0$ are known. $\beta_{\mathrm{r}} < 0$ is known, and $R_0$ denotes the radius of the target stars.

In general, typical path constraints for aerocapture include altitude $h$, heat rate $Q$, load $a$, and dynamic pressure $\bar{q}$,

$$h \geqslant h_{\min} \tag{2}$$

$$\dot{Q} = k_Q \sqrt{\rho / R_{\mathrm{n}}} V^3 \leqslant \dot{Q}_{\max} \tag{3}$$

$$a = \sqrt{L^2 + D^2} / m \leqslant a_{\max} \tag{4}$$

$$\bar{q} = \rho V^2 / 2 \leqslant \bar{q}_{\max} \tag{5}$$

where $k_Q$, $R_{\mathrm{n}}$ are known constants, and $h_{\min}$, $\dot{Q}_{\max}$, $a_{\max}$ and $\bar{q}_{\max}$ denote the limits of altitude, heat rate, load, and dynamic pressure constraints, respectively.

In this paper, we consider the aerocapture guidance law design problem with consideration of the path constraints (2)-(5) for the dynamics (1); furthermore, we prove convergence and carry out numerical simulations under uncertainties to verify the validity of the designed guidance laws.

## 2 Unified numerical predictor-corrector guidance based on characteristic model

Unified numerical predictor-corrector guidance based on characteristic model means that the designed guidance law satisfies both path constraints and guidance targets. In this paper, by transforming the path constraints into the differential function of altitude, characteristic models including the apogee deviation and the altitude differential are established, andthe guidance law is designed.

### 2.1 Establishing the differential equations associated with path constraints

In [12], differential equations related to heat rate, load and dynamic pressure are established. In addition to the above path constraints, aerocapture needs to satisfy altitude constraint (2). The differential equation associated with the altitude constraint is given to provide a basis for the unified guidance law. Discretizing the altitude equation in eq. (1), we have

$$h(t+T) = h(t) + TV \sin \gamma$$

where $T$ denotes the sampling period. In order to achieve the altitude limit

$$h(t+T) = h(t) + TV \sin \gamma \geqslant h_{\min}$$

we have

$$\sin \gamma \geqslant \frac{h_{\min} - h}{VT} \tag{6}$$

Combining the results of [12] for path constraints on heat rate, load and dynamic pressure, by differentiating the path constraints (3)-(5), followed by linear interpolation and prediction, it can be obtained separately:

$$\sin \gamma \geqslant \frac{\dot{Q}_{\max} - \dot{Q}\left[1 - 3TD/(mV)\right]}{\beta_{\mathrm{r}} \dot{Q} V T / (2h_0)} \tag{7}$$

$$\sin\gamma \geqslant \frac{a_{\max} - a\left[1-2TD/(mV)\right]}{\beta_r aVT/h_0} \tag{8}$$

$$\sin\gamma \geqslant \frac{\bar{q}_{\max} - \bar{q}\left[1-2TD/(mV)\right]}{\beta_r \bar{q}VT/h_0} \tag{9}$$

For the inequality(6)-(9), let

$$\sin\gamma_{\mathrm{ref}}^{h} = \max\left\{\sin\gamma, \frac{h_{\min} - h}{VT}\right\} \tag{10}$$

$$\sin\gamma_{\mathrm{ref}}^{\dot{Q}} = \max\left\{\sin\gamma, \frac{\dot{Q}_{\max} - \dot{Q}\left[1-3TD/(mV)\right]}{\beta_r \dot{Q}VT/(2h_0)}\right\} \tag{11}$$

$$\sin\gamma_{\mathrm{ref}}^{a} = \max\left\{\sin\gamma, \frac{a_{\max} - a\left[1-2TD/(mV)\right]}{\beta_r aVT/h_0}\right\} \tag{12}$$

$$\sin\gamma_{\mathrm{ref}}^{\bar{q}} = \max\left\{\sin\gamma, \frac{\bar{q}_{\max} - \bar{q}\left[1-2TD/(mV)\right]}{\beta_r \bar{q}VT/h_0}\right\} \tag{13}$$

Due to the need to satisfy multiple constraints mentioned above at the same time, let

$$\sin\gamma_{\mathrm{ref}} = \max\left\{\sin\gamma_{\mathrm{ref}}^{h}, \sin\gamma_{\mathrm{ref}}^{\dot{Q}}, \sin\gamma_{\mathrm{ref}}^{a}, \sin\gamma_{\mathrm{ref}}^{\bar{q}}\right\} \tag{14}$$

Considering further the dynamics(1), let

$$\dot{h}_{\mathrm{ref}} = V\sin\gamma_{\mathrm{ref}} \tag{15}$$

Remark 1: When it is far from the path constraints, it can be obtained from eq. (6)-(15) that

$$\sin\gamma_{\mathrm{ref}} = \sin\gamma$$

$$\dot{h}_{\mathrm{ref}} = V\sin\gamma$$

$$e_{\dot{h}} = \dot{h} - \dot{h}_{\mathrm{ref}} = 0 \tag{16}$$

Remark 2: When approaching the path constraint, if $e_{\dot{h}} \to 0$ we have $h - h_{\min} \to 0$, $\dot{Q} - \dot{Q}_{\max} \to 0$, $a - a_{\max} \to 0$, or $\bar{q} - \bar{q}_{\max} \to 0$.

### 2.2 Characteristic modeling

The aerocapture numerical predictor-corrector guidance is implemented by first integrating for the guidance dynamics(1) to an altitude of 150 km at the edge of the atmosphere, using the current state as the initial value. Then from the algebraic relations obtained from the two-body problem

$$a_e = \frac{\mu}{2\mu/r_e - V_e^2} \tag{17}$$

$$r_a^e = a_e\left(1 + \sqrt{1 - \frac{V_e^2 r_e^2 \cos^2\gamma_e}{\mu a_e}}\right) \tag{18}$$

calculate the apogee $r_a^e$ reached by the spacecraft after aerocapture, where $x_e = [r_e \quad V_e \quad \gamma_e]^{\mathrm{T}}$ denotes the state of exiting atmosphere. The error at the apogee is

$$e_a = r_a^e - r_a^* \tag{19}$$

where $r_a^*$ denotes the expected apogee. In particular, the integration of the state variable $\gamma$ requires the guidance law $\sigma$. At the current moment $t$, we only know the guidance law $\sigma(t)$ that was designed before the current moment, but the guidance law after the current moment required for the integration is unknown. To solve this problem, a constant bank angle strategy is used, i. e., the guidance law after the current moment is needed to obtain the integral, and the guidance law $\sigma(t)$ at the current moment is used. By integrating the 5th equation of eq. (1) without considering rotation,

$$\gamma_e(t) = f_\gamma(t) + g_\gamma(t)\cos\sigma(t) \tag{20}$$

where

$$f_\gamma(t) = \gamma(t) + \int_t^{h=150\ \mathrm{km}} \left(\frac{V}{r} - \frac{1}{Vr^2}\right)\cos\gamma \,\mathrm{d}\tau$$

$$g_\gamma(t) = \int_t^{h=150\ \mathrm{km}} \frac{L}{mV}\mathrm{d}\tau$$

Theorem 1: If the sampling period is selected in accordance with the theory of characteristic model, the characteristic model for united numerical predictor-corrector guidance can be established as slow time-varying differential equations:

$$e_a(k) = e_a(k-1) + g_a(k)\Delta u(k) \tag{21}$$

$$e_{\dot{h}}(k) = f_{\dot{h}1}(k)e_{\dot{h}}(k-1) + f_{\dot{h}2}(k)e_{\dot{h}}(k-2) + g_{\dot{h}}(k)\Delta u(k) \tag{22}$$

where $e_{\dot{h}}(k)$ and $e_a(k)$ are given in(16)and(19) respectively,

$$\Delta u(k) = \cos\sigma(k) - \cos\sigma(k-1)$$

$f_{\dot{h}1}(k) \in \left[\frac{5}{3}, \frac{29}{15}\right], f_{\dot{h}2}(k) \in \left[-\frac{14}{15}, -\frac{2}{3}\right]$

and $g_a(k)$ and $g_{\dot{h}}(k)$ are bounded.

Proof: We first prove that eq. (21) holds. Denote the state of the atmospheric fringe such that $e_a = 0$ as

$$\boldsymbol{x}_e^* = [r_e^* \quad V_e^* \quad \gamma_e^*]^T$$

It can be seen that $e_a$ is a function of $\boldsymbol{x}_e = [r_e \quad V_e \quad \gamma_e]^T$ by eq. (17) and (18). Thus, we take a linear expansion of $e_a$ at $x_e^*$:

$$e_a(t) = \frac{\partial e_a}{\partial r_e}[r_e(t) - r_e^*] + \frac{\partial e_a}{\partial V_e}[V_e(t) - V_e^*] + \frac{\partial e_a}{\partial \gamma_e}[\gamma_e(t) - \gamma_e^*]$$

Substituting eq. (20), we get

$$e_a(t) = \frac{\partial e_a}{\partial r_e}[r_e(t) - r_e^*] + \frac{\partial e_a}{\partial V_e}[V_e(t) - V_e^*] + \frac{\partial e_a}{\partial \gamma_e}\gamma_e^* + \frac{\partial e_a}{\partial \gamma_e} f_\gamma(t) + \frac{\partial e_a}{\partial \gamma_e} g_\gamma(t)\cos\sigma(t) \tag{23}$$

By discretization of eq. (23), we have

$$e_a(k) = \frac{\partial e_a}{\partial r_e}[r_e(k) - r_e^*] + \frac{\partial e_a}{\partial V_e}[V_e(k) - V_e^*] - \frac{\partial e_a}{\partial \gamma_e}(k)\gamma_e^* + \frac{\partial e_a}{\partial \gamma_e} f_\gamma(k) + \frac{\partial e_a}{\partial \gamma_e} g_\gamma(k)\cos\sigma(k)$$

Further at step $k-1$, we have

$$e_a(k-1) = \frac{\partial e_a}{\partial r_e}[r_e(k-1) - r_e^*] + \frac{\partial e_a}{\partial V_e}[V_e(k-1) - V_e^*] - \frac{\partial e_a}{\partial \gamma_e}(k-1)\gamma_e^* + \frac{\partial e_a}{\partial \gamma_e} f_\gamma(k-1) + \frac{\partial e_a}{\partial \gamma_e} g_\gamma(k-1)\cos\sigma(k-1)$$

Differentiate the above two equations:

$$e_a(k) = e_a(k-1) + g_a(k)\Delta u(k) + \Delta e(k)$$

where

$$g_a(k) = \frac{\partial e_a}{\partial \gamma_e} g_\gamma(k)$$

$$\frac{\partial e_a}{\partial \gamma_e} = \frac{V_e^2 r_e^2 \sin 2\gamma_e}{2\mu\sqrt{1 - \frac{V_e^2 r_e^2 \cos^2\gamma_e}{\mu a_e}}}$$

$$\Delta e_a(k) = \frac{\partial e_a}{\partial r_e}(r_e(k) - r_e^*) + \frac{\partial e_a}{\partial V_e}[V_e(k) - V_e^*] - \frac{\partial e_a}{\partial \gamma_e}(k)\gamma_e^* + \frac{\partial e_a}{\partial \gamma_e} f_\gamma(k) - \frac{\partial e_a}{\partial r_e}[r_e(k-1) - r_e^*] - \frac{\partial e_a}{\partial V_e}[V_e(k-1) - V_e^*] + \frac{\partial e_a}{\partial \gamma_e}(k-1)\gamma_e^* - \frac{\partial e_a}{\partial \gamma_e} f_\gamma(k-1) + \left[\frac{\partial e_a}{\partial \gamma_e} g_\gamma(k) - \frac{\partial e_a}{\partial \gamma_e} g_\gamma(k-1)\right]\cos\sigma(k-1) \tag{24}$$

Since the states are bounded in the engineering problems, it follows that $g_a(k)$ is bounded. From eq. (24) it can be seen that each quantity in $\Delta e_a(k)$ is the difference between the states of two continuity sampling periods, and thus it's a small quantity. From the characteristic model theory[13], a slow time-varying characteristic model can be built when the sampling period is small:

$$e_a(k) = e_a(k-1) + g_a(k)\Delta u(k)$$

In the following we prove eq. (22). Taking differential and substituting eq. (1), it can be obtained that

$$\dot{e}_h = F_{\dot{h}} + G_{\dot{h}} \cos\sigma \tag{25}$$

where

$$F_{\dot{h}} = -\frac{D}{m}\sin\gamma - \frac{\mu}{r^2} + \frac{V^2\cos^2\gamma}{r} - \ddot{h}_{ref}$$

$$G_{\dot{h}} = \frac{L\cos\gamma}{m}$$

and $\ddot{h}_{ref}$ is the differential of eq. (15). By analysing eq. (14) and (15) it can be seen that when $\dot{e}_h \neq 0$, $\sigma(t)$ is not explicitly included in $\ddot{h}_{ref}$; when $\dot{e}_h = 0$, $\dot{e}_h$ is not included in the guidance law in the following design, so the guidance law term $\sigma(t)$ in $\ddot{h}_{ref}$ is not considered here. The method of [15] is introduced below by letting

$$\bar{F}_h = \begin{cases} -\dfrac{F_{\dot{h}}}{\dot{e}_h}, & \dot{e}_h \neq 0 \\ -\dfrac{\partial F_{\dot{h}}}{\partial \dot{e}_h}, & \dot{e}_h = 0 \end{cases} \tag{26}$$

Taking discretization of eq. (25), it can be obtained

$$e_h(k)=\bar{F}_h(k)e_h(k-1)+g_h(k)\cos\sigma(k) \tag{27}$$

where

$$\bar{f}_h=(1-T\bar{F}_h),g_h=TG_h$$

For the engineering problems, $\bar{f}_h$ and $G_h$ are bounded. From the characteristic model theory[13], by selecting $T$ such that

$$T\,|\bar{F}_h|\in\left[\frac{1}{15},\frac{1}{3}\right] \tag{28}$$

we have

$$\bar{f}_h\in\left[\frac{2}{3},\frac{14}{15}\right] \tag{29}$$

By eq. (27), at step $k-1$, we have

$$\begin{aligned}e_h(k-1)=&\bar{f}_h(k-1)e_h(k-2)+\\&g_h(k-1)\cos\sigma(k-1)\end{aligned} \tag{30}$$

Subtract eq. (30) from eq. (27):

$$\begin{aligned}e_h(k)=&f_{h1}(k)e_h(k-1)+f_{h2}(k)e_h(k-2)+\\&g_h(k)\Delta u(k)+[g_h(k)-g_h(k-1)]\cos\sigma(k-1)\end{aligned} \tag{31}$$

where

$$f_{h1}(k)=1+\bar{f}_h(k),f_{h2}(k)=-\bar{f}_h(k-1)$$

By (29) it can be seen that

$$f_{h1}(k)\in\left[\frac{5}{3},\frac{29}{15}\right],f_{h2}(k)\in\left[-\frac{14}{15},-\frac{2}{3}\right]$$

Further, it can be shown that the last term in eq. (31) is the difference between the states of two continuity sampling periods, and thus it's a small quantity. From the characteristic model theory[13], a slow time-varying characteristic model can be established when the sampling period is small:

$$\begin{aligned}e_h(k)=&f_{h1}(k)e_h(k-1)+f_{h2}(k)e_h(k-2)+\\&g_h(k)\Delta u(k)\end{aligned}$$

Combining the above analysis, it can be shown that the conclusion of the theorem holds. The proof is complete.

Remark 3: The choosing method of the sampling period in Theorem 1 is given in [16]. In [16], the time scale of a nonlinear system and the way its sampling period is chosen for nonlinear systems are defined. Specifically for the nonlinear system (25), $\bar{F}_h$ given in eq. (26) is its time scale, and the sampling period can be chosen in accordance with eq. (28).

Remark 4: From the characteristic model theory[13], the characteristic models and the bounds on their parameters established in Theorem 1 are consistent with the basic characteristics of first-and second-order characteristic models.

Remark 5: Characteristic models cannot be modelled in the guidance problem by means of a direct discretization of the guidance dynamics, and a method for characteristic modelling of the guidance problem is presented in Theorem 1. In fact, in the control problem, the error used in the control design is the output of the dynamics at the current moment, and therefore, we can build a characteristic model of the control problem by performing a direct discretization of the dynamics[13]. Unlike the control problem, the error used in the guidance design, instead of being the output of the guidance dynamics at the current moment, is the integration over the guidance dynamics to obtain the error used in the guidance design. The guidance problem for aerocapture is even more complex, and in addition to the integration described above, it is also necessary to take algebraic operations to obtain the apogee error for use in the guidance law design. Therefore, we cannot establish the characteristic models in the guidance problem by directly discretizing the guidance dynamics. In Theorem 1, a method for characteristic modelling of the guidance problem is presented.

### 2.3 Designing guidance laws

In this section we design united numerical predictor-corrector guidance laws based on characteristic model. Let

$$\{\boldsymbol{\theta}(k) \mid \boldsymbol{\theta}(k)=[g_a \quad f_{\dot{h}1} \quad f_{\dot{h}2} \quad g_{\dot{h}}]^{\mathrm{T}}, f_{\dot{h}1} \in \left[\frac{5}{3},\frac{29}{15}\right], f_{\dot{h}2} \in \left[-\frac{14}{15},-\frac{2}{3}\right], g_a, g_{\dot{h}} \in [0.003,1]\} \tag{32}$$

$$\{\hat{\boldsymbol{\theta}}(k) \mid \hat{\boldsymbol{\theta}}(k)=[\hat{g}_a \quad \hat{f}_{\dot{h}1} \quad \hat{f}_{\dot{h}2} \quad \hat{g}_{\dot{h}}]^{\mathrm{T}}, \hat{f}_{\dot{h}1} \in \left[\frac{5}{3},\frac{29}{15}\right], \hat{f}_{\dot{h}2} \in \left[-\frac{14}{15},-\frac{2}{3}\right], \hat{g}_a, \hat{g}_{\dot{h}} \in [0.003,1]\} \tag{33}$$

$$\boldsymbol{\Phi}(k)=\begin{bmatrix}\Delta u(k) & 0 & 0 & 0\\ 0 & e_{\dot{h}}(k-1) & e_{\dot{h}}(k-2) & \Delta u(k)\end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{y}(k)=[e_a(k)-e_a(k-1) \quad e_{\dot{h}}(k)]^{\mathrm{T}}$$

where the bounds of $\hat{g}_{\mathrm{a}}$ and $\hat{g}_{\dot{h}}$ are chosen according to the characteristic model theory[13], and $\hat{\boldsymbol{\theta}}(k)$ denotes the identification result of $\boldsymbol{\theta}(k)$. United numerical predictor-corrector guidance laws based on characteristic model consists of three parts.

1) Choosing the initial values:

$$\hat{\boldsymbol{\theta}}(0)=[0.5 \quad 2 \quad -1.5 \quad 0.5]^{\mathrm{T}}$$

$$\boldsymbol{\Phi}(0)=\boldsymbol{0}_{4\times 2}$$

2) Identifying the parameters: firstly, the gradient method or least squares method is used to identify the parameters, and then the identification results are projected into eq. (33).

3) Designing the guidance law: first design guidance laws based on the characteristic models (21) and (22),

$$\Delta u(k)=-\frac{L_1}{\lambda_1+\hat{g}_a(k)}e_a(k-1)-\frac{L_2\hat{f}_{\dot{h}1}(k)}{\lambda_2+\hat{g}_{\dot{h}}(k)}e_{\dot{h}}(k-1) \tag{34}$$

where $0<L_1, L_2<1, \lambda_1, \lambda_2>0$, then take integration of eq. (34),

$$u(k)=u(k-1)+\Delta u(k) \tag{35}$$

where the initial value $u(1)$ is a known guidance quantity.

## 2.4 Convergence analysis

Theorem 2: Consider the united numerical predictor-corrector guidance law based on characteristic model eq. (34) and (35). There exist control parameters $L_1, L_2, \lambda_1, \lambda_2$ such that for the characteristic models (21) and (22) the closed-loop system is convergent.

Proof: let

$$\bar{e}_{\dot{h}1}(k-1)=e_{\dot{h}}(k-2)$$

$$\bar{e}_{\dot{h}2}(k-1)=e_{\dot{h}}(k-1)$$

$$F_{u1}=-\frac{L_1}{\lambda_1+\hat{g}_a(k)}, F_{u2}=-\frac{L_2\hat{f}_{\dot{h}1}(k)}{\lambda_2+\hat{g}_{\dot{h}}(k)} \tag{36}$$

Substituting eq. (34) into characteristic models (21) and (22) gives the closed-loop system

$$\begin{bmatrix}e_{\mathrm{a}}(k)\\ \bar{e}_{\dot{h}1}(k)\\ \bar{e}_{\dot{h}2}(k)\end{bmatrix}=\begin{bmatrix}1+g_{\mathrm{a}}F_{u1} & 0 & g_{\mathrm{a}}F_{u2}\\ 0 & 0 & 1\\ g_{\dot{h}}F_{u1} & f_{\dot{h}2} & f_{\dot{h}1}+g_{\dot{h}}F_{u2}\end{bmatrix}\times\begin{bmatrix}e_{\mathrm{a}}(k-1)\\ \bar{e}_{\dot{h}1}(k-1)\\ \bar{e}_{\dot{h}2}(k-1)\end{bmatrix} \tag{37}$$

Since the characteristic model is slowly time-varying, the analysis is carried out here using "the solidified coefficient method" which uses the eigenvalues to determine the stability. For the error $e_{\mathrm{a}}$, its main feedback quantity is generated by $F_{u1}e_{\mathrm{a}}$, so the analysis ignores the value of row 1, column 3 in eq. (37). In this case, the closed-loop system (37) can be divided into two parts:

$$z-(1+g_{\mathrm{a}}F_{u1})=0 \tag{38}$$

$$z^2-(f_{\dot{h}1}+g_{\dot{h}}F_{u2})z-f_{\dot{h}2}=0 \tag{39}$$

It is clear from the Juli judgement[17] that the condition for stability is

$$-2<g_{\mathrm{a}}F_{u1}<0 \tag{40}$$

$$1+f_{\dot{h}1}-f_{\dot{h}2}>-g_{\dot{h}}F_{u2} \tag{41}$$

or

$$1-f_{\dot{h}1}-f_{\dot{h}2}>g_{\dot{h}}F_{u2} \tag{42}$$

Combining the bounds on the parameters (32) and the expression (36) it is known that there exist control parameters $L_1, L_2, \lambda_1, \lambda_2$

such that eq. (40) and (41) hold. The proof is complete.

## 3 Numerical simulations

Numerical simulations are carried out using Mars spacecraft as the object of research. The mass of the spacecraft is 2 500 kg, the velocity increment is 2 077 m/s, and the specific impulse is 310 s. The guidance, controland filtering methods[6] are designed. In [18], the autonomous navigation problem for the aerocapture approach and capture segments are investigated, achieving positional and velocity accuracies of 50 m and 5 m/s respectively. In the simulation, we consider this navigation bias with atmospheric entry and aerodynamic uncertainties.

Through the simulation, aerocapture is achieved and path constraints are satisfied. The obtained simulation results in table 1 show the effectiveness of the integrated guidance method proposed in this paper.

Table 1 Simulation results of characteristic model based unified numerical predictor-corrector guidance for Mars aerocaprute

| Uncertainties/% | | | | $V_{sum}$/ (m·s$^{-1}$) | Fuel conservation/% | $r$/km | Error/% |
|---|---|---|---|---|---|---|---|
| Entry angle | Density | Lift coefficient | Drag coefficient | | | | |
| +3 | +50 | +20 | +20 | 51.15 | 98.00 | 0.857 | 0.038 1 |
| +3 | +50 | +20 | −20 | 51.90 | 97.98 | 5.95 | 0.264 |
| +3 | +50 | −20 | −20 | 50.81 | 98.02 | 3.54 | 0.157 |
| +3 | −50 | +20 | +20 | 48.90 | 98.09 | 4.04 | 0.18 |
| +3 | −50 | +20 | −20 | 47.28 | 98.15 | 1.39 | 0.061 9 |
| +3 | −50 | −20 | +20 | 47.53 | 98.14 | −0.158 | −0.007 04 |
| +3 | −50 | −20 | −20 | 39.28 | 98.46 | −20.28 | −0.90 |
| −3 | +50 | +20 | +20 | 45.32 | 98.23 | 0.945 | 0.042 |
| −3 | +50 | +20 | −20 | 45.72 | 98.33 | −4.53 | −0.201 |
| −3 | +50 | −20 | +20 | 41.69 | 98.37 | −8.99 | −0.4 |
| −3 | +50 | −20 | −20 | 39.58 | 98.45 | −13.19 | −0.59 |
| −3 | −50 | +20 | +20 | 43.34 | 98.31 | 1.67 | 0.074 4 |
| −3 | −50 | +20 | −20 | 44.52 | 98.26 | 8.62 | 0.383 |

## 4 Conclusion

Considering the aerocapture guidance problem, a predictor-corrector guidance method based on unified characteristic model is proposed with convergence analysis in this paper. Joint numerical simulations of guidance and control are carried out under consideration of navigation bias, atmospheric entry and aerodynamic uncertainties, demonstrating the effectiveness of the designed method. The design and analysis methods presented can be generalised to general aerodynamic entry guidance and therefore have good application prospects.

References

[1] LONDON H S. Change of satellite orbit plane by

aerodynamic maneuvering [J]. Journal of the Aerospace Sciences, 1962, 29(3): 323-332.

[2] CRUZ M I. The aerocapture vehicle mission design concept: AIAA 79-0893 [R]. Reston: AIAA, 1979.

[3] BRYANT L E, TIGGES M A, IVES D G. Analytic drag control for precision landing and aerocapture: AIAA 1998-4572 [R]. Reston: AIAA, 1998.

[4] VINH N X, JOHNSON W R, LONGUSKI J M. Mars aerocapture using bank angle modulation: AIAA 2000-4424 [R]. Reston: AIAA, 2000.

[5] ROZANOV M, GUELMAN M. Aeroassisted orbital maneuvering with variable structure control [J]. Acta Astronautica, 2008, 62(1): 9-17.

[6] LU P, CERIMELE C J, TIGGES M I, et al. Optimal aerocapture guidance [J]. Journal of Guidance, Control and Dynamics, 2015, 38(4): 553-565.

[7] JITS R Y, WALBERG G D. Blended control, predictor-corrector guidance algorithm: an enabling technology for Mars aerocapture [J]. Acta Astronautica, 2004, 54(6): 385-398.

[8] MOSELEY P E. The Apollo entry guidance: a review of the mathematical development and its operational characteristics: 69-FMT-791 [R]. Houston: TRW, 1969.

[9] LU P. Predictor-corrector entry guidance for low lifting vehicles [J]. Journal of Guidance, Control, and Dynamics, 2008, 31(4): 1067-1075.

[10] ZIMMERMAN C, DUKEMAN G, HANSON J. Automated method to compute orbital reentry trajectories with heating constraints [J]. Journal of Guidance, Control, and Dynamics, 2003, 26(4): 523-529.

[11] BRAUN R D, POWELL R W. Predictor-corrector guidance algorithm for use in high-energy aerobraking system studies [J]. Journal of Guidance, Control, and Dynamics, 1992, 15(3): 672-678.

[12] LU P. Entry guidance: a unified method [J]. Journal of Guidance, Control, and Dynamics, 2014, 37(3): 713-728.

[13] WU H H, HU J. Method and application for characteristic model [M]. Beijing: National Defense Press, 2022 (in Chinese).

[14] RECASENS J J, CHU Q P, MULDER J A. Robust model predictive control of a feedback linearized system for a lifting-body re-entry vehicle: AIAA 2005-6147 [R]. Reston: AIAA, 2005.

[15] MENG B, ZHAO Y B, MU J J. A characteristic modeling method of error-free compression for nonlinear systems [J]. Control Theory and Technology, 2021, 19: 375-383.

[16] MENG B, WU H X. On characteristic modeling of a class of flight vehicles' attitude dynamics [J]. Science China Technological Sciences, 2010, 53(8): 2074-2080.

[17] DUAN G R. Linear system theory [M]. Haerbin: Haerbin Institute of Technology Press, 2004 (in Chinese).

[18] XIONG X K. On autonomous navigation technology for Mars exploration [D]. Beijing: China Academy of Space Technology, 2023 (in Chinese).

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